Analysis of the height dependence of temperature and density of Knudsen gases at thermal equilibrium

Research work produced from January 1996 to January 1997
Presented by Rolf Freitag, University of Bremen, Germany
English translation

1997-01-15

Contents

1 Introduction

In a very thin atmosphere (Knudsen gas [1]) the interaction energy (=potential energy) between the part systems planet and Knudsen gas is of the same order of magnitude as the thermal energy of the Knudsen gas. Because the molecules of the Knudsen gas do move virtually on trajectory parabolas the atmosphere can't be separated into independent subsystems.

One consequence of this is that the Knudsen gas has no maxwellian velocity distribution, because when e. g. the velocity distribution of the gas atoms is beeing measured, which do start vertical from the floor, thus arrive vertical at the height h, their velocity square is reduced by $2 \cdot g \cdot h$. Therefore the velocity distribution is shifted to lower velocities namely non-thermal, i. e. because of the free fall it is no maxwellian distribution anymore.

Additionally by the shift to lower velocities the gas temperature is decreased, because the temperature is proportional to the average velocity square.

In the following paper at first the height dependence of the velocity probability distribution is calculated also for other angles. From this follows, by integration above the phase space, both density and temperature at height h. The results are afterwards compared with monte-carlo simulations.
For simplicity for the most part only monatomic Knudsen gases are considered, because for not monatomic gases the results would give nothing new.

[1] I. e. the gas molecules do collide with the walls of the container, inclusive floor and ceiling, more often than with other gas molecules.

2 Mainpart

2.1 The kinetic gas theory of Knudsen gases

2.1.1 The height dependence of the velocity probability distribution

For simplicity only a monatomic Knudsen gas in a cubic container of length a in a homogeneous force field of acceleration g is considered (see Fig.1).

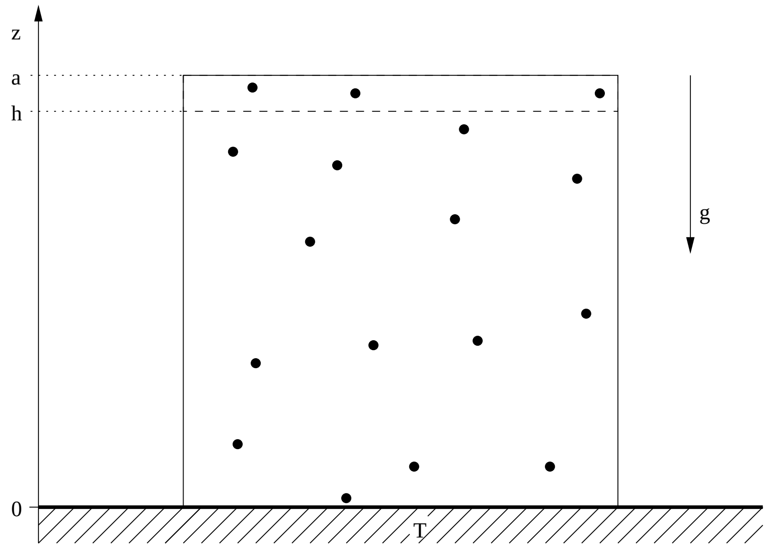

Abb. 1: Knudsen gas in a cubic container

The walls and the ceiling is thermal insulating and the floor has the temperature T because of the thermal reservoir under the container. The floor and the ceiling do reflect the gas atoms diffuse, i. e. angle independent. The walls do reflect the gas atoms mirroring (elastic) and therefore do have no influence to the temperature and density of the gas.

The floor and the to the outside thermal isolated ceiling are therefore in a local thermodynamic equilibrium with the gas, they do have the temperature of the gas there.
This model is realistic, because for example Hg-atoms are reflected diffusive from glass walls and reflecting from rock salt surfaces [Heise-63].

The gas atoms do move only on trajectory parabolas between the floor and the ceiling. Therefore an atom which has started from the floor with velocity v_0 has the velocity $v = \sqrt{v_0^2 - 2 \cdot g \cdot h}$ at height h. Solved to the velocity on the floor we do get $v_0 = \sqrt{v^2 + 2 \cdot g \cdot h}$.

For a gas, which has the maxwellian distribution on the floor (h=0), from this follows the velocity distribution which is modified by the free fall of the gas atoms:

$$\varphi(v,h) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2 \cdot k \cdot T}\right)^{\frac{3}{2}} \cdot (v^2 + 2 \cdot g \cdot h) \cdot e^{-\frac{(v^2+2\cdot g\cdot h)\cdot m}{2\cdot k\cdot T}} \frac{dv_0}{dv} \tag{1}$$

where the last factor, the Jacobian of v, is equal to $2 \cdot v \cdot 1/(2 \cdot \sqrt{v^2 + 2 \cdot g \cdot h})$.

This not the complete velocity probability density distribution function at height h, because the full dependence on angle and height as well as normalisation is not included.

For the general case of the oblique throw it is necessary to take into account that only molecules with [2]

$$v_0 \geq \frac{\sqrt{2 \cdot g \cdot h}}{\cos(\theta)} \tag{2}$$

can reach the height h. This can be achieved by the factor $\Theta\left(\frac{\sqrt{2\cdot g\cdot h}}{\cos(\theta)}\right)$.

This leads to the next velocity probability density distribution function at height h:

$$\varphi_\theta(v,h) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2 \cdot k \cdot T}\right)^{\frac{3}{2}} \cdot (v^2+2\cdot g\cdot h) \cdot e^{-\frac{(v^2+2\cdot g\cdot h)\cdot m}{2\cdot k\cdot T}} \cdot \Theta\left(v - \sqrt{\frac{2 \cdot g \cdot h}{\cos(\theta)^2} - 2 \cdot g \cdot h}\right) \cdot \frac{v}{\sqrt{v^2 + 2 \cdot g \cdot h}} \tag{3}$$

The Heaviside function can be simplified to $\Theta\left(v - \sqrt{2 \cdot g \cdot h} \cdot \tan(\theta)\right)$ and will appear in the following integrals only as the lower integration limit.

By normalisation we finally do get the normalised velocity probability density distribution function at height h:

$$\frac{\varphi_\theta(v,h)}{\int_0^{2\pi} \int_0^{\pi/2} \int_0^{\infty} \varphi_\theta(v,h) \cdot \sin(\theta) dv d\theta d\varphi} \equiv \varphi_\theta(v,h)_N \tag{4}$$

At the integration over the latitude it has been take into account that the integration is done in spherical coordinates and that therefore the Jacobian is equal to $\sin(\theta)$. Generally here we also do have an angle dependence of the gas atoms which do start from the floor.

[2] θ =latitude, angle of the trajectory parabola to the vertical at the floor; φ is the longitude.

By the shift to lower velocities and the Theta function the velocity probability density distribution function is completely different from the maxwellian distribution.
In the following chapters the consequences of this are investigated.

2.1.2 The density of the Knudsen gas

2.1.2.1 Calculation of the density of one trajectory parabola

The vertical probability density of an atom on a trajectory parabola with the maximum height M is equal to the spended time between height h and h+dz, thus

$$\frac{dt(h)}{dz} = \frac{1}{v_z(h)} = \frac{1}{\sqrt{2g(M-h)}} \tag{5}$$

This density has dimension s/m. The particle density follows from this by multiplication with s.

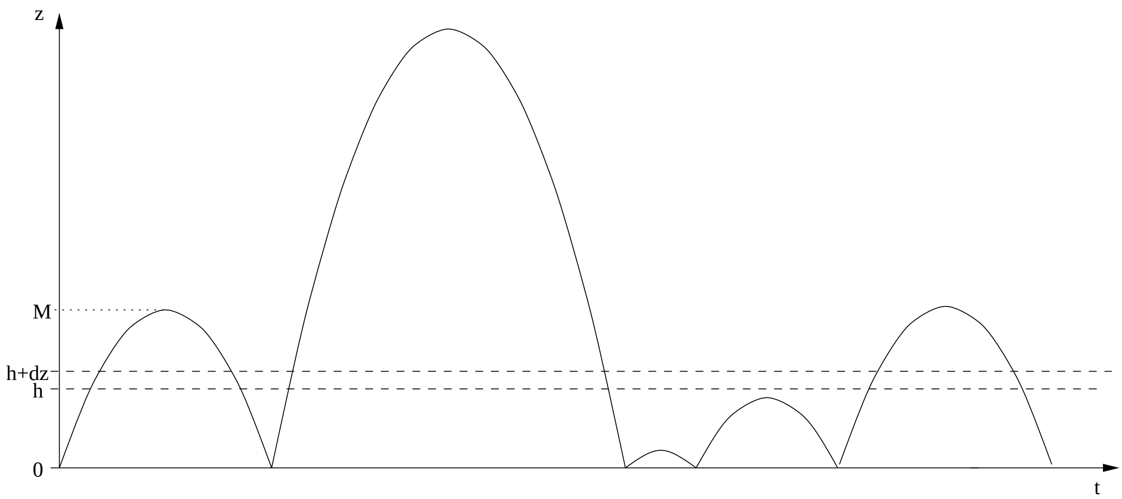

Abb. 2: Space-time diagram for the gas density at height h.

The maximum height of the trajectory parabola is:

$$M = \frac{v_{0,z}^2}{2 \cdot g} = \frac{v_0^2 \cdot \cos(\theta)^2}{2 \cdot g} \tag{6}$$

At this height the density has a singularity which vanishes when the Heisenberg uncertainty principle or only finite volumes are taken into account.
For the x- and y-direction the same is valid but with constant velocity, therefore constant density and is consequently trivial.

2.1.2.2 Calculation of the Knudsen gas density

The Knudsen gas density results in integration over the trajectory parabolas on which the atoms do move in the Knudsen gas. Because the density for a trajectory parabola has been calculated in the previous chapter, the normalised integration over all trajectory parabolas (=integration over the phase space) remains.

In an ensemble of trajectory parabolas and at thermal equilibrium the contribution of a trajectory parabola is proportional to their density (eq. 5) and their formation probability (due to the maxwellian distribution).

The gas density is therefore the integral over the phase space probability density $\varrho(x, y, z, p_y, p_y, p_z) = \varphi_\theta(v, z)_N$ multiplied with the probability density at height z=h. Due to equation 5 and the maxwellian distribution (eq. 13 with $v = v(h = 0) = v_0$, because the formation is on the floor) we do obtain the density:

$$\begin{aligned}
&\frac{4}{\sqrt{\pi}}\left(\frac{m}{2\cdot k\cdot T}\right)^{\frac{3}{2}}\int_0^{\pi/2}\int_{\sqrt{2\cdot g\cdot h\cdot\tan(\theta)}}^{\infty} v_0^2\cdot e^{-\frac{v_0^2\cdot m}{2\cdot k\cdot T}}\cdot\frac{1}{\sqrt{2\cdot g(M-h)}}\cdot\sin(\theta)\cdot dv_0\cdot d\theta\\
&=\frac{4}{\sqrt{\pi}}\left(\frac{m}{2\cdot k\cdot T}\right)^{\frac{3}{2}}\cdot\int_0^{\pi/2}\int_{\sqrt{2\cdot g\cdot h\cdot\tan(\theta)}}^{\infty}(v^2+2\cdot g\cdot h)\cdot e^{-\frac{(v^2+2\cdot g\cdot h)\cdot m}{2\cdot k\cdot T}}\\
&\cdot\frac{1}{\sqrt{2\cdot g(M-h)}}\frac{v}{\sqrt{v^2+2\cdot g\cdot h}}\cdot\sin(\theta)\cdot dv\cdot d\theta
\end{aligned}\tag{7}$$

Because the integrand is independent of φ and we only want to know the relative density, for simplicity the integration over φ as well as normalisation have been omitted.

The substitution

$$\frac{(v^2+2\cdot g\cdot h)\cdot\cos(\theta)^2}{2g}=M\tag{8}$$

simplifies equation 7 to:

$$\frac{4}{\sqrt{\pi}}\left(\frac{m}{2\cdot k\cdot T}\right)^{\frac{3}{2}}\cdot\int_h^{\infty}\int_0^{\pi/2}\frac{\sqrt{2}\cdot g^{3/2}\cdot M\cdot e^{-\frac{M\cdot g\cdot m}{k\cdot T\cdot\cos(\theta)^2}}}{\cos(\theta)^4\cdot\sqrt{M-h}}\sin(\theta)\cdot dM\cdot d\theta\tag{9}$$

The result of the calculation is the relative vertical density:

$$\varrho(h)=e^{-\frac{m\cdot g\cdot h}{k\cdot T}}-\frac{1}{2}Ei\left(-\frac{m\cdot g\cdot h}{k\cdot T}\right)\tag{10}$$

Here Ei is the integral exponential function. Through this the difference to the barometrical height formula is significant, because it has a logarithmic singularity at height h = 0 m.

This singularity vanishes when the van der Waals force between the gas atoms and the floor is take into account because on trajectory parabolas which are nearly parallel to the ground (i. e. $\theta \approx \frac{\pi}{2}$) the van der Waals force is stronger than $m \cdot g$. Thus for the trajectory parabolas there is a maximum angle θ_{max} which is a little less than $\pi/2$. A finite surface roughness has the same effect. For the record, anyhow the density in the Knudsen gas decreases faster than the barometric height formula indicates.

One consequence is that the average potential energy of a Knudsen gas atom is less than $\frac{kT}{2}$. This also follows from the energy conservation law, because on the trajectory parabolas the sum of the potential energy and the kinetic energy is at average $\frac{kT}{2}$. Because both energys are greater than zero they are both smaller than $\frac{kT}{2}$ (triangle inequality).
Because the degree of freedom height z is included in the Hamilton function linear and not quadratic, this is no contradiction to the principle of equipartition (which is valid only in classical statistical mechanics).

2.1.2.3 Possible measurement of the Knudsen gas density

The density can directly be quantified only by the measurement of the number of the gas atoms in a volume at height h. Because Knudsen gases do scatter light, e-beams etc. very weakly, it is hardly possible to measure the density by measuring the scattering or absorption.
Therefore for measuring the density it is suitable to measure the radiation of a strong radioactive gas, for example Radon, height dependent because the activity is proportional to the density.

2.1.3 The Knudsen gas temperature

The temperature of a monatomic Knudsen gas is, due to the kinetic gas theory, the average kinetic energy of the gas atoms times 1/2k, thus ([Reif-87]):

$$T = \frac{m \cdot \overline{v^2}}{4 \cdot k} \tag{11}$$

With polyatomic Knudsen gases the vibrations and rotations have to be taken into account.

If the temperature is not estimated out of the average velocity square of a gas stream but instead from an ensemble, then the temperature is [Kuch-89]:

$$T = \frac{m \cdot \overline{v^2}}{3 \cdot k} \tag{12}$$

Here we have a difference of 3/4, which is caused by that the gas stream is gas density * velocity (=v). Therefore the velocity distribution of the atoms which do hit a thermometer is the maxwellian velocity distribution times v (and a normalisation constant [Miller-55]).

Because nearly all gas temperature measurement methods do use only the average kinetic energy of the gas molecules which do hit the thermometer, in the following it is assumed that only the first of these formulas is beeing used.

2.1.3.1 Calculation of the Knudsen gas temperature

The Knudsen gas temperature results in integration over the trajectory parabolas on which the atoms do move in the Knudsen gas. At this in equation 11 for the mean average velocity square the velocity square of the trajectory parabola at height h has to be set.
In an ensemble of trajectory parabolas and at thermal equilibrium the contribution of an trajectory parabola is proportional to their formation probability (due to the maxwellian distribution).
Therefore the normalised integration over all trajectory parabolas (=integration over the phase space) remains.
Here we have to take into account that the temperature is not only a quality of the ensemble but a quality of the gas atoms which do hit the thermometer.
Therefore instead of the simple the modified maxwellian distribution has to be use. Instead of

$$\varphi(v) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2 \cdot k \cdot T}\right)^{\frac{3}{2}} \cdot v^2 \cdot e^{-\frac{v^2 \cdot m}{2 \cdot k \cdot T}} \tag{13}$$

we have to use

$$\varphi_m(v) = 2 \left(\frac{m}{k \cdot T}\right)^2 \cdot v^3 \cdot e^{-\frac{v^2 \cdot m}{2 \cdot k \cdot T}} \tag{14}$$

[Miller-55].
The reason for this, that the atoms do have the maxwellian distribution on the ground but the gas stream, which is emitted from the floor, has the modified maxwellian distribution, is that the stream density is equal to density * velocity ($= v_0$, see also [Reif-87]).
Therefore the modified maxwellian distribution has, compared with the maxwellian distribution, additional the factor v and another normation constant.

For this reason the gas temperature is the integral over the phase space probability $\varrho_m(x, y, z, p_y, p_y, p_z) = \varphi_{\theta,m}(v, z)_N$ (due to eq. 4 and 14) times the single temperature at height z=h (eq. 11), so:

$$T(h) = \frac{m \cdot \overline{v^2}}{4 \cdot k} = \frac{m}{4 \cdot k} \int_0^{2\pi} \int_0^{\pi/2} \int_{\sqrt{2 \cdot g \cdot h \cdot \tan(\theta)}}^{\infty} v^2 \cdot \varphi_{\theta,m}(v, h)_N \cdot \sin(\theta) \cdot dv \cdot d\theta \cdot d\varphi \tag{15}$$

The substitution

$$\frac{(v^2 + 2 \cdot g \cdot h) \cos(\theta)^2}{2g} = M \tag{16}$$

simplifies equation 15 to:

$$\frac{m}{4k}\frac{\int_h^\infty\int_0^{\pi/2}\frac{(2\cdot M)^2\cdot g^3\cdot e^{-\frac{M\cdot m\cdot g}{k\cdot T\cdot\cos(\theta)^2}}}{\cos(\theta)^6}\cdot\sin(\theta)\cdot dM\cdot d\theta}{\int_h^\infty\int_0^{\pi/2}\frac{2\cdot M\cdot g^2\cdot e^{-\frac{M\cdot m\cdot g}{k\cdot T\cdot\cos(\theta)^2}}}{\cos(\theta)^4}\cdot\sin(\theta)\cdot dM\cdot d\theta}-\frac{2\cdot m\cdot g\cdot h}{4\cdot k} \tag{17}$$

The result is:

$$T(h)=\frac{T}{4}\cdot\frac{3\cdot\sqrt{m\cdot g\cdot h\cdot k\cdot T\cdot\pi}\cdot erfc\left(\sqrt{\frac{m\cdot g\cdot h}{k\cdot T}}\right)e^{\frac{m\cdot g\cdot h}{k\cdot T}}-8\cdot k\cdot T-2\cdot m\cdot g\cdot h}{\sqrt{m\cdot g\cdot h\cdot k\cdot T\cdot\pi}\cdot erfc\left(\sqrt{\frac{m\cdot g\cdot h}{k\cdot T}}\right)e^{\frac{m\cdot g\cdot h}{k\cdot T}}-2\cdot k\cdot T}-\frac{2\cdot m\cdot g\cdot h}{4\cdot k} \tag{18}$$

This temperature distribution in a Xenon Knudsen gas at a floor temperature of 293 K is shown in fig. 3.

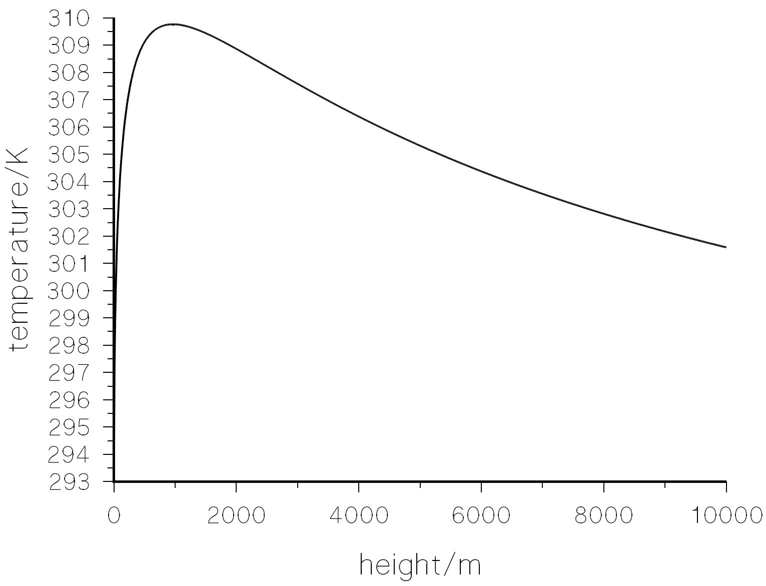

Abb. 3: The temperature in Kelvin of Xenon Knudsen gas on earth at a floor temperature of 293 K plotted over the height h in m.

Because the parameters m, g and h are in the equation as a product, for monoatomic gases the result is qualitatively the same, the maximum temperature is the same and the limit value for $m \cdot g \cdot h$ towards infinity is T.

It is remarkable that here are two converse effects:
The first effect is that the ceiling can only be reached from fast atoms with

$v_0 \geq \frac{\sqrt{2 \cdot g \cdot h}}{\cos(\theta)}$, which means that only hot gas does reach the ceiling.
The second effect is that by the gravity the gas atoms do get slower on the way to the ceiling and therefore are at the ceiling cooler than on the floor.
Altogether cooled hot gas reaches the ceiling and without the calculation above, it is not clear if the ceiling is cooler or warmer than the floor at thermal equilibrium.
Because of the Liouville theorem it is known that the phase space volume of the gas, which starts from the floor and reaches the ceiling, is constant. Because that gas reaches the ceiling with a lower space density, from the Liouville theorem follows that the impulse density is higher at the ceiling. But because the impulse is decreased by the gravity and only the relative energy-rich part of the gas can reach the ceiling this does not help at estimating if the ceiling at thermal equilibrium is cooler or warmer than the floor.
The two converse effects do explain why the temperature first increases, reaches a maximum and than decreases.

2.1.3.2 Possible measurement of the Knudsen gas temperature

Because the heat conductivity of a Knudsen gas is very low a buildup due to Fig. 4 has to be shielded against outside temperature gradients and the in the floor and ceiling used high precision temperature sensors must have a power consumption of maximum 1 μW, to avoid that they do measure only their self-heating. Therefore in Fig. 4 temperature measurement quartzes are drawn into the floor and ceiling. The resonance frequency of these quartzes is proportional to the temperature.

Furthermore the floor and ceiling should have a low emission ratio (e. g. by smooth and gold plated surfaces), because otherwise the heat conduction by heat radiation can be stronger than the heat conduction by the Knudsen gas.

In the buildup the walls are omissive because a contamination of the fine vacuum of approx. 1 Pa can be simplest prevented by a very small gas stream.

The heat conduction power between two parallel plates of area a in Knudsen gas is [Heise-63]:

$$P_{therm} = \frac{a \cdot n \cdot m \cdot \overline{c}}{8} \left(\overline{c_1^2} - \overline{c_2^2} \right) \tag{19}$$

By inserting the temperatures T and $T(h)$ as well as the mean free path $\lambda = 1/(\sqrt{2} \cdot n \cdot \pi \cdot d^2)$ due to [Ber-74] we get:

$$P_{therm} = \frac{3 \cdot a \cdot k^{1.5} \cdot \sqrt{T(h) + T}}{8 \cdot \sqrt{2 \cdot m} \cdot \pi \cdot \lambda \cdot d^2} (T(h) - T) \tag{20}$$

Here d is the atom diameter, p_{therm} the heat conduction power, with which the floor does heat the equal temperature ceiling.

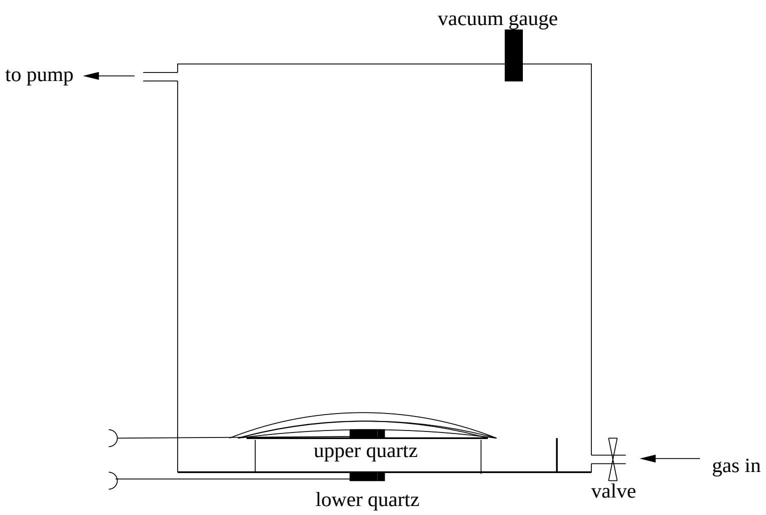

Abb. 4: Buildup for measuring the Knudsen gas temperature.

For example Xenon with h = 1 cm, $\lambda = 2$ cm, $T = 293\ K$, $a = 1\ m^2$ and $g = 9.81\ m/s^2$ gives only $P_{therm} = 5.1\mu$W.

Because of this small power the calculated temperature difference $T(h) - T$ of 0.149 K needs some minutes for setting up.
The ceiling plate is therefore upside isolated by multiple aluminium foils and glass fibre fabric (so called superisolation, see [Kohlrausch-86]) and supported by three to four bad heat conducting sustainer (e. g. 0.4 mm stainless steel canulas).
The temperature measurement is done by measuring the frequency of the two oscillators, which are connected to the quartzes. For a better temperature resolution it is wise to mix both frequencies with a reference frequency and to measure only the difference frequencies. Because a frequency counter shows only the absolute value, the reference frequency must be a little lower or higher than both quartz frequencies. Otherwise it is only possible to measure only absolute value temperature differences and not temperature differences.

2.2 The height dependence of the heat radiation due to the general relativity theory

Because the scattering cross section of the photon-photon scattering is generally very small, the photon gas is known in the broader sense as a Knudsen gas.

Therefore for the energy fluxes between two parallel plates of emissivity ε in a homogeneous gravitation field we have an energy flux diagram as shown in Fig. 5

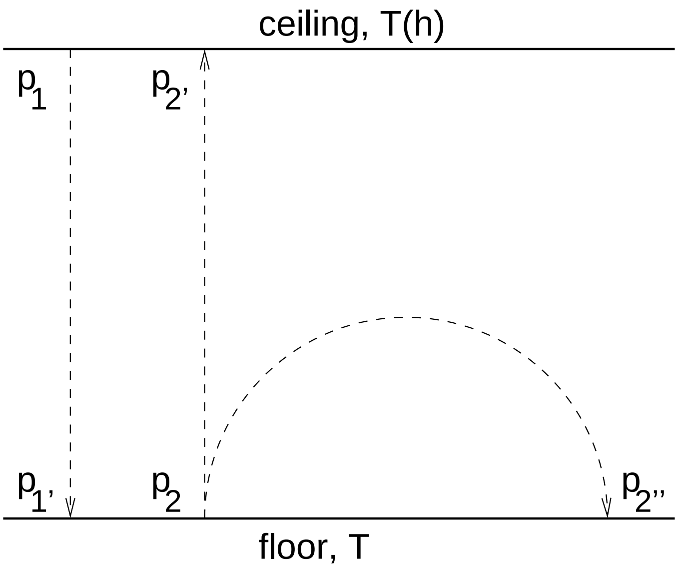

Abb. 5: The energy fluxes of heat radiation between two parallel plates in a homogeneous gravitation field.

The photons are moving along zero geodaetas [Stephani-91]

$$\frac{d^2x^\mu}{d\lambda^2} = -\Gamma^n_{ab}\frac{dx^\theta}{d\lambda}\frac{dx^\lambda}{d\lambda} \tag{21}$$

with

$$g_{\theta\beta}\frac{dx^\theta}{d\lambda}\frac{dx^\beta}{d\lambda} = 0 \tag{22}$$

In the gravitation field of a spherical star nearly hyperbolas [Duschek-65], therefore in the homogeneous gravity field nearly trajectory parabolas and have, in contrast to the trajectory parabolas, no energy dispersion. For the first proximity in the following the calculations are done with trajectory parabolas with v=c. Because there is no energy dispersion, the photons do reach the ceiling for that $v_z = c \cdot \cos\Theta \geq \sqrt{2gh}$ is true. That is that the angle between the trajectory parapola tangent at the floor and the vertical is smaller than

$$\arccos\sqrt{\frac{2\cdot g\cdot h}{c^2}}\quad . \tag{23}$$

The floor radiates with the power density

$$p_2=\sigma\cdot\varepsilon\cdot T^4 \tag{24}$$

The ceiling radiates analogous with the power density

$$p_1=\sigma\cdot\varepsilon\cdot T(h)^4 \tag{25}$$

For simplification of the calculations and elemination of multiple reflections in the following ε of floor and ceiling is equal to one.
Because not all photons, which are emitted from the floor, do reach the ceiling, a part of the power density of the floor,

$$p_{2''}=\frac{1}{\pi}\int\limits_0^{2\pi}\int\limits_{\arccos\sqrt{\frac{2\cdot g\cdot h}{c^2}}}^{\pi/2} p_2\cdot\sin(\theta)\cdot\cos(\theta)d\theta d\varphi=2\int\limits_{\arccos\sqrt{\frac{2\cdot g\cdot h}{c^2}}}^{\pi/2}\sigma\cdot T^4\cdot\sin(\theta)\cdot\cos(\theta)d\theta \tag{26}$$

goes directly back to the floor. Therefore at the ceiling we do have less power density than on the floor.

The factor $\sin(\theta)$ is the Jacobian in spherical coordinates and the factor $2\cdot\cos(\theta)$ is the light density distribution (due to the second Lambert cosine law).

The photons, which are emitted from the floor and reach the ceiling, are emitted with a power density of $p_{2'}-p_{2''}$. At the ceiling, they are red shifted by

$$\frac{1}{\sqrt{1+\frac{2\cdot g\cdot h}{c^2}}}=\frac{1}{q} \tag{27}$$

The photon energy is therefore at the ceiling only 1/q of the photon energy on the floor. The photon power density is decreased by the red shift to 1/q.

This red shift is for example known as red shift from stars [Ber-78].
By this the heat radiation of the floor is not heat radiation at the ceiling; it is not according to the plank radiation formula.

Besides the red shift also the time dilatation has to be included, because as is generally known a clock on a mountain is faster than a clock in a valley. The time dilatation is at the ceiling smaller by the factor

$$\frac{1}{\sqrt{1+\frac{2\cdot g\cdot h}{c^2}}}=\frac{1}{q} \tag{28}$$

compared with the floor. The number of photons per time unit, which do reach the ceiling, is therfore only 1/q of the photons which do start from the floor to the ceiling. The power density is therefore decreased to 1/q by the time dilatation.

Altogether we have three effects which solitary do decrease the power density which reaches the ceiling from the floor. So it is clear that the ceiling at thermal equilibrium is colder than the floor and the calculation of the ceiling temperature by using the balance equation remains.

The outcome of the balance equation is

$$p_1 = p_{2'} \tag{29}$$

so

$$\frac{2}{q^2} \int_0^{\arccos\sqrt{\frac{2\cdot g\cdot h}{c^2}}} \sigma \cdot T^4 \cdot \sin(\theta) \cdot \cos(\theta) d\theta = \sigma \cdot T(h)^4 \tag{30}$$

After substitution of q and solving for T(h) we have:

$$T(h) = T \cdot \sqrt[4]{\frac{2}{1 + \frac{2\cdot g\cdot h}{c^2}} \int_0^{\arccos\sqrt{\frac{2\cdot g\cdot h}{c^2}}} \sin(\theta) \cdot \cos(\theta) d\theta} \tag{31}$$

For T = 293 K, $g = 9.81\ m/s^2$ and h = 1 m the outcome of this is a temperature difference of only $1.6 \cdot 10^{-14}$ K.

The calculated effect is therefore not measurable on earth but on neutron star surfaces it gives a heating from the cosmic background radiation which is stronger as closer the star surface is near the Schwarzschild radius.
Furthermore at temperature measurements of compact stars it has to be taken into account that as the star surface gets closer the Schwarzschild as stronger the star radiation is not thermic because of the time dilatation, the red shift and the photons, which fall back on the star.

3 Measurements with a model system

3.0.1 Estimation of the height dependence of the Knudsen gas density by Monte-Carlo simulations with collisions

Mainly for validation of the calculated Knudsen gas density the density was also estimated by Monte-Carlo simulations with elastic collisions of spheres. In addition the density was estimated for the case where collisions between the gas atoms can not be neglected.
The result of the simulations is a validation of equation 10 for the Knudsen gas. For gases with many collisions the resulting density gets closer to the barometrical height formula as the number of collisions increases.

3.0.2 Estimation of the height dependence of the Knudsen gas temperature by Monte-Carlo simulations with collisions

Mainly for validation of the calculated Knudsen gas temperature the temperature was also estimated by Monte-Carlo simulations with elastic collisions of spheres. In addition the temperature was estimated for the case where collisions between the gas atoms can not be neglected.
The result of the simulations is a validation of equation 18 for the Knudsen gas. For gases with many collisions the resulting temperature gets closer to the floor temperature as the number of collisions increases.

Because the gas heat conductivity is proportional to the pressure at low pressures and constant at middle and high pressure, there is a maximum of the thermal power of the gas (heat flux from the floor to the ceiling times $(T(h)-T)$) at equal temperature of floor and ceiling or constant small temperature difference. As by the radiometer effect [Heise-63] the effect (heat conduction power at equilibrium) is greatest, when the average mean free path is approximately the distance h between floor and ceiling.

4 Summary and outlook

As the analytical calculations and the simulations do show, an acceleration field in a Knudsen gas at thermal equilibrium causes a density which decays stronger than exponentially and a height dependent temperature.
In theory this is not extraordinary because a Knudsen gas in an acceleration field can not be separated into independent part systems and can therefore not characterised with canonical collectivities (see [Brenig-92] S. 10).
In practice the results are extraordinary, because the decomposability into independent part systems is a necessary and sufficient prerequisite for the additivity of the part energies and entropies and therefore a necessary prerequisite for the Gibbs postulate and the second law of thermodynamics (see [Brenig-92] Chapter 10.2).
Because of this the results of this work, for example the height dependent temperature in the Knudsen gas, are not in contradiction to the second law of thermodynamics. The results do only show that simplifications like characterisation with canonical collectivities or the second law of thermodynamics are generally not practicable if a prerequisite is missing.

An important point is the isotropic emission of the gas atoms from the floor, which was a prerequisite of the calculations above. Generally the floor emitts more tricky, with a orientation dependent intensity.

In theory a rough surface should emit the gas atoms due to the second Lambert cosine law, which means with a cosine shape intensity and orientation independent flux density, as pointed out in the chapter about the photon gas. This is seen in good approximation in reality, for example with nitrogen molecular beams which are directed on metal or glass surfaces [Hur-57].

But this is true only in approximation and not exact. Furthermore there are notable discrepancies, e. g. on crystal surfaces where interference of the matter waves (=gas atoms) can be seen [Lth-93].
If the cosine orientation dependence is put into the equations for the calculations of the density and temperature in the Knudsen gas, they show the results for the dense gas, which means exponentially with the height decreasing density and constant temperature. Because the calculations with the photon gas were done with this factor, nothing has to be changed there.
Because in dense gases the atoms do collide against a wall due to the second Lambert cosine law and the gas exchange between two different gas layers is due to the second lambert cosine law [Reif-87] the results have been expected.
For the record, anyhow a floor which does not exactly due to the second Lambert cosine law causes that the temperature of the Knudsen gas above is not constant and that the density does not decrease exponentially. The same is true for a not homogenous acceleration field, which is the case at the example with the thin planet atmosphere, which is not in a homogenous but in a spherically symmetric

field.
But these two effects are small.

In the calculations and simulations for simplicity only a one-piece plane and parallel to the ground ceiling was used.
With other ceilings, for example a multi-piece ceiling, comprising convex and concave surfaces, the angle distribution of the atoms which do hit a pice of the ceiling can be selected by the shape of the piece of the ceiling because the velocity distribution function (eq. 4 resp. $\varphi_{\theta,m}(v,z)_N$ due to eq. 4 and 14) depends on the angle and height. An example is a sphere which is hanging under a plane ceiling, because the sphere does receive the atoms from the floor nearly isotropic, without the cosine factor from the second Lambert cosine law even if the floor emitts due to the second Lambert cosine law.
This has been verified with simulations. For a sphere in a Knudsen gas above a due to the second Lambert cosine law emitting floor the temperature is the temperature due to do 18.

In theory the temperature differences, calculated and verified by simulations, can be used without problems for conversion of environment heat into other sorts of energy, e. g. electricity. The calculated little heat conduction powers and relatively small temperature differences in the Knudsen and photon gas can hardly technically be used because even with greater accelerations, e. g. from ultracentrifuges, only a power density of few watt per cubic meter can be achieved.
Another way would be other gases, e. g. electron gases in semiconductors, a miniaturisation, series and parallel connection because a relative thin gas can be achieved (average mean free path $<$ container length) by reduction of the gas density or the container length and by using many containers with series and parallel connection the temperature difference and heat conduction power can be increased.

It is remarkable that there are other ways to convert environment heat into other sorts of energy or temperature differences.
An example is an isolated vertical metal cable in a plasma, for example a Wolfram cable isolated with ThO_2 in a Cs plasma at 2000 to 3000 K. By the gravity the inside the metal free movably electrons are pulled down with the gravity force $m_e \cdot g$ and in stationary equilibrium without current this causes an electric field of strength E_g, which pulls the electrons up so that there is a floating (no net movement) of the electrons: $m_e \cdot g = E_g \cdot q_e$.
By measuring this field, on accelerated conductors, C. R. Tolman and T. D. Steward have measured the electron mass in 1916 [Ber-71].
Such a field can not come into existence in a plasma because an electric field would be exhausted by the free movable negative and positive ions.
When the metal cable in the plasma is not isolated at the ends, the electric field in the cable causes a current from the lower cable end, through the plasma, to the upper cable end. The current of this current generator can be used, as any

other electric current, to supply an electric circuit.
But technically this is hard to do because the gravity on earth causes an electric field of only $E_g = g \cdot m_e/q_e = 55,8\,pV/m$ in metals.

Even using many of this current sources in series and parallel connection does not help, because the low field strength requires a giant length which causes a high internal resistance. Because of the high internal resistance only very little electric powers can be achieved.

Literature

[Ber-71] Bergmann-Schaefer: **Lehrbuch der Experimentalphysik**, Band II, 6. Auflage, 1971, S. 483-484.

[Ber-74] Bergmann-Schaefer: **Lehrbuch der Experimentalphysik**, Band I, 9. Auflage, 1974, S. 653.

[Ber-78] Bergmann-Schaefer: **Lehrbuch der Experimentalphysik**, Band III, 7. Auflage, 1978, S. 962.

[Brenig-92] Brenig, Wilhelm: **Statistische Theorie der Wrme, Bd. 1: Gleichgewichtsphnomene**, Berlin, New York, 3. Aufl., 1992.

[Carmesin-96] Carmesin, Hans-Otto: **Grundlagen zur Relativit"atstheorie**, Berlin, 1996, S. 59.

[Duschek-65] Duschek, Adalbert; Hochrainer, August: **Grundzge der Tensorrechnung in analytischer Darstellung**, Wien, New York, 2. Aufl., III. Teil, 1965, Seite 271.

[Heise-63] Heise, Ruth: **Elementare Einf"uhrung in die kinetische Gastheorie**, Leipzig, 1963.

[Hur-57] Hurlbut, F. C.: **Studies of Molecular Scattering at the Solid Surface**, Journal of Applied Physics, Volume 28, Number 8, 1957, page 844 f..

[Jackson-82] Jackson, John David: **Klassische Elektrodynamik**, Berlin, New York, 2. Aufl., 1982, Seite 266.

[Kohlrausch-86] Kohlrausch, Friedrich: **Praktische Physik**, Band 3, Stuttgart, 1986, S. 105.

[Kuch-89] Kuchling, Horst: **Taschenbuch der Physik**, Frankfurt/Main, 1989, 12. Auflage, S. 252.

[Miller-55] R. C. Miller and P. Kusch: **Velocity Distributions in Pottasium and Thallium Atomic Beams**, Physical Review, Volume 99, Number 4, 1955, Page 1314-1321.

[Lth-93] Lth, H.: **Surfaces and Interfaces of Solids**, 2nd Edition 1993, ISBN 3-540-56840-9, page 251.

[Messiah-91] Messiah, Albert: **Quantenmechanik**, Band 1, Berlin, New York, 2. Aufl., 1991, Seite 352.

[Reif-87] Reif, Frederick: **Statistische Physik und Theorie der Wrme**, Berlin, New York, 3. Aufl., 1987, S. 322.

[Stephani-91] Stephani, Hans: **Allgemeine Relativittstheorie**, Berlin, 1991, S. 18.